Newton's grazing-angle ring table follows his own rule, not the physics

A preregistered comparison of the obliquity table in the Opticks with exact thin-film theory, from perpendicular light to 90 degrees

Zain Dana Harper

Version 1.0, 9 November 2026. Finding F14 in the findings ledger. Signal (tier B).

Abstract

In Book II of the Opticks, Newton gives a table of the air thickness at a coloured ring for thirteen viewing angles, and beside it a rule he fitted to the table. We put every row beside exact thin-film optics and beside his rule. From perpendicular light to 60 degrees in the air, his table matches the secant law of modern optics within 0.41 percent. From 65 to 90 degrees every row matches his own 106-means rule within 1 percent, and at 85 degrees his tabled thickness sits 26.7 percent below the secant. Exact theory says the secant holds at every angle below the critical one, so the steep rows follow his rule and not the light. Our reading is that those rows were computed from the rule; the text alone cannot settle it. The same model checks his best-known number: under standard daylight weighted by the eye, the first dark ring falls at 275.7 nm, 3.4 percent below Newton's 1/89000 inch (285.4 nm). All five preregistered predictions held and both controls failed as required.

1 What this means for a reader

Newton's table of rings at slanting light is usually read as a set of measurements. Set beside exact optics, the rows split in two. The shallow rows track the physics, and the steep rows track an arithmetic rule Newton states on the same page. In any old table, a row that agrees with its author's formula more closely than the instrument could measure is more likely computed than observed.

2 Introduction and motivation

Newton's rings are the bright and dark circles seen where a curved lens rests on flat glass. In Book II, Part I of the Opticks [1], Newton measured them with a lens of known curvature and turned ring diameters into thicknesses of the air gap. In Observation 6 he gives the gap at the first dark ring as 1/89000 inch for light near the perpendicular. In Observation 7 he tilts the view and tabulates the thickness at one ring for thirteen values of the obliquity, scaled so that perpendicular light gives 10. He says his values came "partly by measuring" and partly by other means, including two prisms for the steepest angles.

Below the table he states a rule he "seem[s] to gather" from it. We call it the 106-means rule: the thickness goes as the secant of an angle whose sine is the first of 106 arithmetical means between the sines of incidence and refraction, counted from the bigger sine. Modern optics gives a simpler answer, the secant law: the thickness at a fixed ring grows as one over the cosine of the angle in the air.

The two agree at shallow angles and part at steep ones. At 90 degrees in the air the secant is infinite, and Newton's last row reads 122 and a half. The question is which of the two his rows follow, and from which angle on.

The literature check for this paper ran three searches on 9 November 2026. Shapiro's study of Newton's theories of coloured bodies and fits [3] works from Newton's manuscripts and the experiments behind Book II, and is the most likely place for an earlier analysis of this table; we found its catalogue and publisher records and did not read it. Morizot [4] traces how Thomas Young turned Newton's ring thicknesses into the first

wavelengths; we read its abstract. Lee and Lee [5] give open code for the colours of layered thin films from the same multiple-beam sums we use; we read its abstract. None of the searches returned an analysis of the Observation 7 table against exact theory. That is what the searches returned, and it does not show that no such analysis exists.

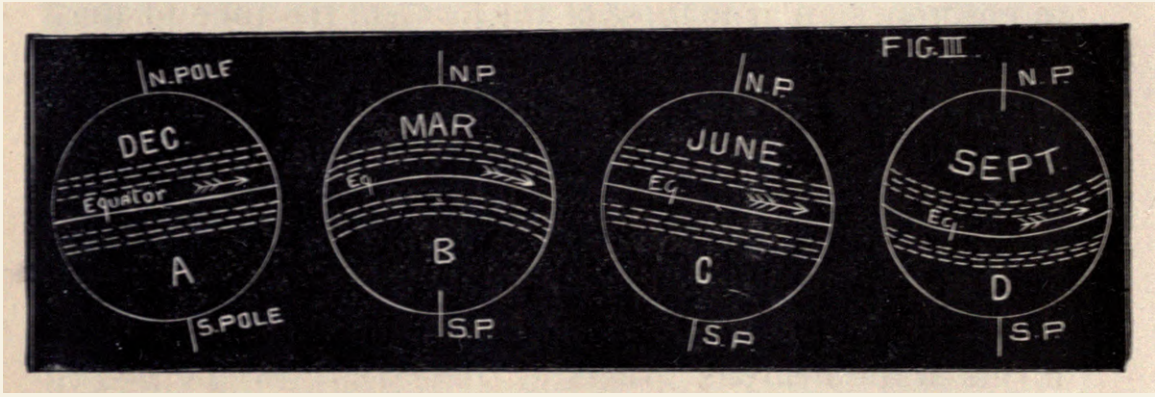


Figure 1. Robert P. Traxler, Fig. III, the apparent path of sunspots across the Sun's disc in December, March, June and September, from The Principles of Mechanics as Applied to the Solar System (San Francisco, 1889) [7]. Traxler used the tilt of these paths to argue for the inclination of the Sun's axis, as part of his own 1889 theory of a repelling force from the Sun, which physics did not adopt. The plate shows the same kind of reasoning this paper tests in Newton: an observation read through the geometry of a slanting view. Public domain; scan from the Internet Archive, University of California Libraries.

3 Preregistration and design

The design was written in a dated preregistration on 28 October 2026. The file states that it was written before the model or the study ran. It was not hash-locked at the time, and its result section was appended to the same file after the run, so the hash given in the reproduction section is of the file as it now stands.

One check came before the file. We had computed the rule by hand at the 90 degree row and found about 122.9 against Newton's 122 and a half. The preregistration says so, and that check is why prediction E exists.

The five predictions, with their bounds:

- A, the first dark ring in white light lies within 5 percent of Newton's 285.4 nm.
- B, the first four bright rings in white light stand as 1 : 3 : 5 : 7 within 3 percent.
- C, for light of one wavelength, 571 nm, the thickness at the first dark ring follows the secant of the air angle within 0.1 percent from 0 to 85 degrees.
- D, Newton's thickness column matches the secant within 1 percent for rows from 0 to 60 degrees.
- E, Newton's thickness column matches his 106-means rule within 1.5 percent at every row, 65 to 90 degrees included.

The file also names two controls that had to fail, given in the controls section. If E holds and D fails beyond 60 degrees, the steep rows follow the fitted rule, and C says the true thickness keeps growing as the secant.

4 Data and provenance

The primary text is the fourth edition of the Opticks, 1730, as transcribed by Project Gutenberg (ebook 33504) [1], with the Newton Project transcription as a second copy [2]. Both are public domain. The thickness column, the glass angles and the air angles of Observation 7 were typed by hand from that text into the study script. Table 2 gives every value that entered.

The white light is the CIE D65 daylight table and the eye's weighting is the CIE 1931 2 degree observer, both

from the official CIE data tables [6]. The glass is N-BK7, with an index of 1.5175 at 571 nm from the model's own refraction table.

5 Methods

The stack and its reflectance

The model is a glass, air gap, glass stack, the geometry of Newton's lens on a plate over the small area of one ring. For light of wavelength λ crossing a gap of thickness d at angle θ_a in the air, write c_a for the cosine of θ_a. The phase delay between successive beams is

$$\delta = \frac{4\pi d c_a}{\lambda} \tag{1}$$

Let r_1 be the amplitude reflection coefficient from glass into air and r_2 that from air into glass, each from the Fresnel equations for one polarisation. Write C for the cosine of δ. Summing every reflected and re-reflected beam gives the Airy reflectance

$$R = \frac{(r_1)^2 + (r_2)^2 + 2 r_1 r_2 C}{1 + (r_1 r_2)^2 + 2 r_1 r_2 C} \tag{2}$$

computed for s and p polarisation and averaged. For this stack $r_2 = -r_1$. That sign change is the half-wave flip, and it makes the point of contact dark.

White light

For white light the reflectance is weighted by the CIE D65 power $S(\lambda)$ and by the eye's brightness curve $y(\lambda)$, from 380 to 780 nm in 1 nm steps:

$$Y(d) = \frac{\sum R(d,\lambda) S(\lambda) y(\lambda)}{\sum S(\lambda) y(\lambda)} \tag{3}$$

Y is the reflected luminance as a share of the incoming. The study stepped d from 0 to 2,000 nm in 0.1 nm steps and read off the minima and maxima of Y. A dark ring is a minimum of Y and a bright ring a maximum.

The obliquity laws

With thickness scaled to 10 at perpendicular light, the secant law is

$$t_{sec} = \frac{10}{c_a} \tag{4}$$

For Newton's rule, let s_a and s_g be the sines of the angles in the air and in the glass that his table lists. The first of 106 arithmetical means between them, counted from the bigger, divides the interval into 107 equal steps, and its value s_φ is the sine of the angle in Newton's secant:

$$s_\varphi = s_a - \frac{s_a - s_g}{107} \tag{5}$$

$$t_{rule} = \frac{10}{\sqrt{1 - (s_\varphi)^2}} \tag{6}$$

At 90 degrees in the air, $s_a = 1$, and eq. 6 stays finite where eq. 4 does not.

Each row's deviation is the absolute difference between Newton's value and the law, divided by the law's value. Prediction C was scored on the model alone: the first dark ring at 571 nm was found at each air angle and its thickness divided by the thickness at perpendicular light.

6 Controls and planted checks

No planted data were used. The check with a known answer is prediction B: textbook theory places the bright

rings at odd multiples of a quarter wavelength, so a model that missed 1 : 3 : 5 : 7 would be wrong before Newton entered.

Two controls had to fail, and both did.

- With the half-wave flip removed, the centre turns bright and the first minimum moves to 137.7 nm, 52 percent from Newton's value. So the dark-ring match depends on the physics the model carries.
- With the thickness column shuffled across rows 200 times, his rule failed the 1.5 percent bound at 7 or more rows in every shuffle, with a median of 12. So the match to the rule depends on the rows being in Newton's order.

One error of ours was caught before grading. The first run gave prediction C a ratio of exactly 1 at every angle, because the dark-ring search started at 1 nm, where reflectance is still rising out of the dark centre, and stopped at once. The search was changed to take the first minimum after the first maximum, and the study was run again. The model was not changed.

7 Results

All five predictions held. Table 1 gives each against its bound.

Table 1. The preregistered predictions and controls against the run.

Prediction	Bound	Measured
A, first dark ring, white light	within 5 percent of 285.4 nm	275.7 nm, 3.4 percent
B, bright rings 1 : 3 : 5 : 7	within 3 percent	1 : 3.005 : 5.031 : 7.070, 1.0 percent
C, model follows the secant to 85 degrees	within 0.1 percent	largest 0.0075 percent
D, table against the secant, 0 to 60 degrees	within 1 percent	largest 0.41 percent
E, table against his rule, all 13 rows	within 1.5 percent	largest 0.99 percent; 0 rows over
Control, no half-wave flip	must miss by over 5 percent	137.7 nm, 52 percent
Control, shuffled column	3 or more rows off the rule	7 or more in all 200

From perpendicular light to 60 degrees in the air, his table matches the secant law of modern optics within 0.41 percent. From 65 to 90 degrees every row matches his own 106-means rule within 1 percent, and at 85 degrees his tabled thickness sits 26.7 percent below the secant. Figure 2 draws the three curves and Figure 3 the deviations row by row. Table 2 lists them.

Table 2. Newton's Observation 7 table beside the two laws. Thickness is 10 at perpendicular light. Glass angles in degrees and minutes as Newton lists them. Deviations in percent of the law's value.

Glass angle	Air angle	His table	His rule	Secant	Off secant	Off rule
0	0	10.00	10.00	10.00	0.00	0.00
6 26	10	10.15	10.15	10.15	0.00	0.01
12 45	20	10.67	10.64	10.64	0.23	0.28
18 49	30	11.50	11.53	11.55	0.41	0.30
24 30	40	13.00	13.02	13.05	0.41	0.18
29 37	50	15.50	15.48	15.56	0.37	0.10
33 58	60	20.00	19.80	20.00	0.00	0.99
35 47	65	23.25	23.31	23.66	1.74	0.26
37 19	70	28.25	28.53	29.24	3.38	0.99
38 33	75	37.00	36.97	38.64	4.24	0.08
39 27	80	52.25	52.29	57.59	9.27	0.07
40 0	85	84.08	84.02	114.74	26.72	0.08
40 11	90	122.50	122.90	none	none	0.33

The 60 degree row equals the secant to the printed precision and sits 0.99 percent from the rule. From 65

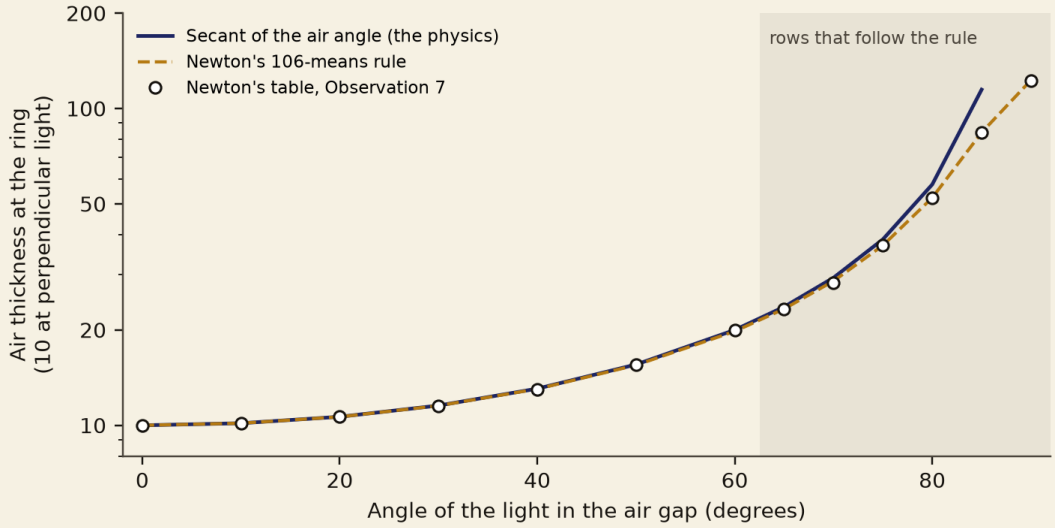


Figure 2. Air thickness at the ring against the angle in the air gap. Circles are Newton's table, the solid line the secant law and the dashed line his 106-means rule. Beyond 65 degrees the circles follow the dashed line. Drawn from the study's result file.

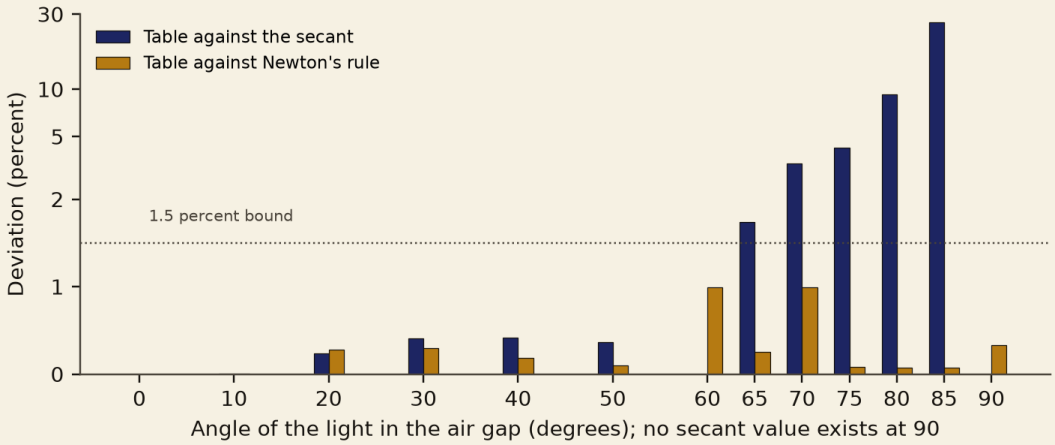


Figure 3. Deviation of each row of Newton's table from the secant law and from his rule, in percent, on a scale that is linear below 2 percent and logarithmic above.

degrees on, the order flips: every row is closer to the rule than to the secant, and the gap to the secant grows with angle, to 9.27 percent at 80 degrees and 26.72 percent at 85.

The model itself keeps the secant. At 571 nm the first dark ring follows the secant of the air angle within 0.0075 percent at every angle from 0 to 85 degrees. The two phase shifts at the faces of the gap cancel at every angle below the critical one, so nothing in the physics bends the curve toward the rule.

Under standard daylight weighted by the eye, the first dark ring falls at 275.7 nm, 3.4 percent below Newton's 1/89000 inch (285.4 nm). The first four bright rings fall at 137.7, 413.8, 692.8 and 973.5 nm, and the first four dark rings at 275.7, 553.2, 832.8 and 1112.7 nm.

8 Discussion

The split in Table 2 has a plain reading. Newton measured the rings at shallow angles, fitted a rule to those

rows, and filled the steep rows from the rule. He wrote that the values came "partly by measuring" and partly by other means, which allows that reading. It is an inference. A rule fitted to the shallow rows would match them, and a row computed from the rule would match it to within the rounding of the printed fractions. Rows that were measured at steep angles and happened to fall on the rule would look the same in the table. Only his manuscripts could tell the two apart.

The rule itself is a good fit to his shallow data and a wrong law. Up to 60 degrees the rule and the secant differ by under 1 percent. Beyond that the rule bends away, and the error reaches 27 percent at 85 degrees.

The 3.4 percent gap at the first dark ring depends on the light and on what counts as dark in a coloured fringe. Our daylight is a modern standard table, and our dark ring is a luminance minimum. Newton's 285.4 nm corresponds to light centred near 571 nm rather than the 551 nm our weighting gives, a shift that sunlight could plausibly give and an eye judging "the darkest Part" of a fringe. We did not test it. Two related studies of ours, on his thick-plate rings and on his colour limits, also find his 1/89000 inch a few percent off, which is a pattern to test and not yet a result.

Young's wavelengths were taken from Newton's ring thicknesses, as Morizot describes [4]. Our result bears on those perpendicular values only through the 3.4 percent gap.

9 Limitations

- Which rows Newton measured and which he computed is our reading. The text cannot settle it, and a historian should check it against his manuscripts and his optical papers.
- The literature search was short. Shapiro's 1993 study [3] was not read and may already cover this table.
- The table was typed by hand from the transcription and was not checked against a scan of the 1730 printing.
- The 3.4 percent gap depends on the light source and on how an eye judges the darkest part of a coloured fringe. The preregistration planned a second check under a solar spectrum; it was not run, and no perceptual model was run.
- The 90 degree row was checked against the rule by hand before the preregistration was written, as that file states.
- The preregistration was not hash-locked at the time it was written.
- Scoring was done by our own code and was not blind.

10 Verified

Verified: on 9 November 2026 the first dark ring values (275.7 nm and 285.39 nm), the ring positions and ratios, the per-row table, rule and secant values, the 0.41 percent and 0.99 percent maxima, the 0.0075 percent secant fit of the model, the 26.7 percent gap at 85 degrees and both control outcomes were re-read from the study's result file. They match the findings ledger and the result section of the preregistration. The glass angles in Table 2 and eqs. 5 and 6 were checked against the study script. Both figures were drawn by a script that reads only that result file. The study was not re-run for this paper. The cited works by Shapiro, Morizot and Lee and Lee were found by searches run on 9 November 2026 and read at the level of catalogue records and abstracts; their full texts were not read.

11 Corrections

- 28 October 2026: the first run's dark-ring search stopped at 1 nm; fixed before grading, model unchanged, as described in the controls section.
- 9 November 2026: the summary written for the site said that at 85 degrees "the rule" sits 26.7 percent below the secant. The result file gives 26.7 percent for the table's value; the rule's own gap at 85 degrees is 26.8

percent. This paper uses the table's value.

- 9 November 2026: first published version.

12 Reproduction

- Data: the Opticks, 1730, Book II, Part I, Observations 6 and 7 (Project Gutenberg 33504); the CIE D65 and CIE 1931 2 degree observer tables.
- Type the thirteen rows of Observation 7: glass angle, air angle and thickness.
- Compare each thickness with eq. 4 and with eqs. 5 and 6.
- For the rings, compute eq. 2 for s and p, average them, weight with eq. 3 over 380 to 780 nm, and find the minima and maxima of Y from 0 to 2,000 nm in 0.1 nm steps. For prediction C, find the first dark ring at 571 nm after the first maximum, at each air angle.
- Controls: repeat the white-light search with the sign of r_2 flipped; shuffle the thickness column 200 times and count rows over 1.5 percent from the rule.
- Preregistration SHA-256, of the file as it now stands with its appended result section: d4104e90dc4a56ea cbee874b19d7cfa1dde3d30326dbf84822d775aab6a0d3fa. No plan hash or run id was recorded for this study; the result file was written on 28 October 2026 at 03:28 UTC and its SHA-256 is 73ca6632eaf61fc7 ab0dec7f10ef8a76f3fb757d5d15583310e1549c5a7d4766.
- Cost: seconds to minutes on one desktop CPU.

13 Appendix A. The preregistered verdicts

Each verdict is a rule written before the run. Values are read from the study's result file.

Table 3. Preregistered verdicts and the rule that decided each one.

Verdict	Deciding rule	Value	Holds
A	first luminance minimum within 5 percent of 285.4 nm	3.4 percent	yes
B	first four maxima within 3 percent of 1 : 3 : 5 : 7	1.0 percent	yes
C	first dark ring at 571 nm within 0.1 percent of the secant, 0 to 85 degrees	0.0075 percent	yes
D	table within 1 percent of the secant, rows 0 to 60 degrees	0.41 percent	yes
E	table within 1.5 percent of the 106-means rule, every row	0.99 percent	yes
Control C1	with no half-wave flip, first minimum misses 285.4 nm by over 5 percent	52 percent	yes
Control C2	shuffled column fails the rule at 3 or more rows	at least 7 of 13	yes

14 Glossary

Each term is defined where it is first used in the text. The same definitions, with the field each comes from and what each excludes, are collected here.

- **Newton's rings**. The bright and dark circles seen where a curved lens rests on flat glass, made by light reflected from the two faces of the thin air gap between them, whose thickness grows with distance from the point of contact. Field: optics. Excludes rings made by diffraction at an edge or an aperture.
- **obliquity**. The angle between the light's path and the perpendicular to the glass; Newton's table gives it twice, once inside the glass and once in the air gap. Field: optics, in Newton's usage. Excludes the tilt of the glass itself, which stays fixed.
- **106-means rule**. Newton's statement that the thickness goes as the secant of an angle whose sine is the first of 106 arithmetical means between the sines of incidence and refraction, counted from the bigger sine. Field: history of optics; Newton's own rule in Opticks, Book II. Excludes the secant law, which uses the air angle alone.

- **secant law**. The statement that the air thickness at which a given ring appears grows as one over the cosine of the light's angle in the air gap, scaled here so that perpendicular light gives 10. Field: thin-film optics. Excludes any rule built from the angle in the glass.
- **preregistration**. A dated written plan for a study, fixed and hash-locked before any result is seen: the question, the data, the statistic, the controls, the forecasts and the kill conditions. Field: research method (metascience). Excludes a plan revised after looking at the outcome, and a protocol written only for the record after the run.
- **control**. A version of the data with the structure under test removed (for example shuffled or pooled), scored the same way as the real data, so the real score has something to beat. Field: statistics and experimental design. Excludes a comparison with a different dataset or with a published number.
- **Airy reflectance**. The share of light power a thin layer reflects when every reflected and re-reflected beam inside it is summed, rather than only the first two. Field: optics; named after George Airy. Excludes the two-beam approximation, and absorption in the glass.
- **s and p polarisation**. The two directions of the light's electric field relative to the plane that holds the ray and the perpendicular to the surface: s across that plane and p within it; unpolarised light is their average. Field: optics. Excludes circular polarisation, which this study does not model.
- **half-wave flip**. The change of sign of the reflected wave at one face of the air gap and not the other, which makes the centre of the rings dark in reflected light. Field: optics. Excludes the phase delay from the path across the gap.
- **CIE D65**. A standard table of the power in average midday daylight at each wavelength, used as the white light in this study. Field: colour science (International Commission on Illumination). Excludes the measured spectrum of any one day's sunlight, and Newton's own light.
- **luminance**. The brightness of light as the standard eye weighs it, computed by weighting power at each wavelength with the CIE 1931 2 degree observer's brightness curve. Field: colour science. Excludes hue and saturation, which the study does not score.
- **findings ledger**. The append-only, dated list of every result we have recorded, numbered F1, F2 and on; corrections are added as new dated lines and old entries are never rewritten. Field: our records. Excludes a curated list of successes.
- **verdict**. The pass or fail of one preregistered rule set against the run's numbers, read off by code. Field: research method (our usage). Excludes an interpretation written after reading the numbers.

15 References

[1] Newton, I. (1730). Opticks: or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light. 4th edition. London. Book II, Part I, Observations 6 and 7. Project Gutenberg ebook 33504.

[2] The Newton Project. The Second Book of Opticks, transcription. University of Oxford. https://newtonproject.ox.ac.uk/view/texts/xml/NATP00038

[3] Shapiro, A. E. (1993). Fits, Passions, and Paroxysms: Physics, Method, and Chemistry and Newton's Theories of Colored Bodies and Fits of Easy Reflection. Cambridge: Cambridge University Press.

[4] Morizot, O. (2026). The wavelength of light as Thomas Young invented it. arXiv:2609.24793.

[5] Lee, D. and Lee, S. (2024). Color simulation for multilayered thin films using Python. arXiv:2412.12828.

[6] International Commission on Illumination (CIE). CIE standard illuminant D65 and CIE 1931 2 degree standard observer colour-matching functions, official data tables.

[7] Traxler, R. P. (1889). The Principles of Mechanics as Applied to the Solar System. San Francisco: C. A. Murdock and Co. Internet Archive item principlesofmech00traxiala.